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Game Theory: when Mathematics meets Systems Psychology

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Abstract

In this paper, our goal is to draw a connection between Systems Psychology and Game Theory. To this aim, we first describe some of the main concepts of these theories, and then we compare them. In particular, we feel very useful and inspiring the connection between the rules of a system and the equilibria of a game. To enlighten the power of this connection, we review a clinical case under the lens of Game Theory. We conclude the work suggesting other possible connections that may exist between these two theories.

1. Introduction

Systems Psychology is a branch of the Psychology that is influenced by Systems Theory (Von Bertalanffy L.,1952). Its main feature consists in assuming that mental diseases depend on the relation that the individual develops with the environment in which she is placed.

The first footprints of this theory can be found in Biology, with the contribution of Von Bertalanffy (Tucker A. W. *et al.*,1959), that introduces the concept of "systems". Later, Von Bertalanffy, in collaboration with biologists, mathematicians and physicists, introduced the General System Theory (Von Bertalanffy L.,1952) that states the key principles that hold for every interactive system. In particular, this work formally defines the concept of system. Another fundamental contribution to Systems Psychology has been given by the Cybernetics, that is "the scientific study of control and communication in the animal and the machine" (Whitaker C. A. *et al.*, 1984). According to this theory, in order for a system to work, it is fundamental that the different components communicate each other and that they are able to adjust themselves through this communication.

In the same period, another theory was developed for modeling the interaction between the components of complex systems: Game Theory. This is a branch of Mathematics that formally models the interaction of different components when the welfare of a component depends not only on her actions, but also on the actions taken by other components.

This area has been initiated by one of the most famous mathematicians in the twentieth century, namely John Von Neumann[1], in a German paper

[1] Besides the foundation of Game Theory, Von Neumann is involved in almost every of the major findings of Mathematics and Physics of the twentieth century, such as the foundation of Computer Science, the development of mathematical foundation of quantum mechanics, the development of the

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published in 1928 (Nash J., 1951) (the English translation appears in Simon F. B., Stierlin H., C. Wynne L. C.,. (1985). However, Game Theory becomes a successful research field only after the 1944, when Von Neumann published the book "Theory of Games and Economic Behavior" with Oskar Morgenstern (Von Neumann J., 1928).
Recently, Game Theory has been used for modeling complex systems in Economics (Aumann R. J. *et al.*,1992; Aumann R. J. *et al.*,1994; Aumann R. J. *et al.*, 2002), Biology (Smith J. M.,1982), Physics (Galam S. *et al.*, 2010), Computer Science (Koutsoupias E. *et al.*; 2009, Levin H. *et al.*, 2008) Sociology (Lorenz J. *et al.*, 2011) and many other disciplines. The success of this theory is also highlighted by the huge number of prizes that have been awarded to game theorists, with twelve Nobel laureates since 1994.
It is then natural to wonder whether Game Theory can contribute also to Systems Psychology. To answer to this question, in this paper we will define some basics concepts of Systems Psychology and Game Theory in Section 2 and 3, respectively. For sake of simplicity, we will avoid to use the standard mathematical language. Rather, we will heavily rely on the usage of simple examples. Finally, in Section 4, we draw a connection between concepts from these two disciplines. Interestingly, we will also show how Game Theory can be sometimes adopted for explaining aspects of therapeutic settings.

2. Systems Psychology Basics

In this section we will introduce some of the main concepts in Systems Theory.
As introduced above, General Systems Theory and Cybernetics have initiated a through study of complex systems, by describing some of the basics concepts and of the main properties behind the work of these systems. Then, the Palo Alto group led by Bateson applied these theories to the analysis of communication and human interaction, and later to the study of psychic diseases.
To understand these contributions, we must start by explicitly formalizing what a system is. A system (from the Greek word systanai, meaning "put together") is an entity whose different parts are interconnected and they interact each other. When this theory is applied to humans, it then suggests that to disclose the mechanisms that are behind the actions and the behavior of an individual, it is not sufficient to focus on this single individual, but it is necessary to investigate about the relation that she maintains. Indeed, Systems Psychology focuses its analysis not on the individual's mind, but on her environment.
The boundaries of this environment are defined open (Von Bertalanffy L., 1969), that is there is a (material or informative) exchange with the external environment, and are opposed to closed systems, where there is no external influence and, consequently, no changes in the system's members.

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atomic bomb

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Every open system enjoys some fundamental propertie (Von Bertalanffy L., 1969): the whole is more than the sum of its parts, that is the system is more complex than the set of its components; holism, that is a change of a component in the system will influence every other component (e.g., in a family, a change of work shifts of the father influences the habits of all the members); negative feedback, that is those processes that enable a system to minimize the perturbations caused by of external or internal influences (e.g., in the above example, every member will continue to require the same level of availability of the father); positive feedback, that is those processes that enable a change or an evolution in the system, whenever the perturbations are introduced by previously received information (e.g., in the above example, members adapt their routines in order to face the limited or changed availability of the father); circularity, that states that if a component influences another one, then also the second can influence the first (e.g., when the son misbehaves, the parents pay more attention to him; this causes the son to behave better, and consequently, the parents to lower their attention; but when the parents arrive to ignore the son, he starts to misbehave again); equifinality, that is the outcomes do not depend on the initial condition of the system, but from the kind of ongoing process and from its parameters. Thus, not only different initial conditions can result in a same outcome, but also similar initial conditions can result in completely different outcomes.

Open systems, such as the family, are led by rules (Von Bertalanffy L., 1969). A relational rule is the application of a kind of relation among the system's members, that turns out to be stable and persistent over time. Note that the members of a system do not need to be conscious about the relational rule they are applying.

Other concepts will be introduced in Section 4, and they will turn out to be useful for establishing a connection between Systems Psychology and Game Theory.

3. Game Theory Basics

Game Theory is used for modeling interactive decision-making processes in complex systems. These systems are modeled as strategic games. A strategic game consists of several ingredients. The first one is given by the set of components of the system, named players. For example, in a family, the players are the husband, the wife, the son, the daughter, and so on.

The next ingredient dictates that each player is equipped with a set of actions, or strategies, that she can take. For example, if we would like to model a couple in which a job is offered to both members, then both players (i.e., the male and the female) have two available strategies: either to accept the job or to refuse it in order to take care of the family.

The set of players and their strategies define the possible outcomes of a strategic game, i.e., the profiles of chosen strategies (one strategy for eachplayer). Roughly speaking, the outcomes of a game are all possible states in which the system can be found, or, alternatively, all possible decisions that can be taken at the end of the decision-making process. For

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the couple example, the possible outcomes are either that both the members accept, or both refuse, or only one of them accepts.

The last ingredient of strategic games then assumes that different players can like different outcomes. Consider again the couple example. We may have that both prefer that at least one accepts the job (because it helps the financial welfare of the couple); similarly, we may have that both prefer that at least one takes care of the family; finally, each of them would like to accept the job for professional gratification. This complex situation can be represented by assuming that each player has an utility function, that evaluates how much that player likes each of the outcomes: for example, for the male "accepting while the other refuses" is the most preferred outcome, "both accepting" is the second most preferred one, "refusing while the other accepts" the third and "both refusing" the less preferred, and similarly for the female. One can depict graphically these preferences as described in Figure 1.

Summarizing, a strategic game is defined by the set of players, the set of strategies that each player can play, and the utility that each player receives from the possible outcomes of the game (where the outcomes are all possible combinations of players' strategies).

The main assumption in Game Theory is that players are utility-maximizer, that is they want maximize their own utility, and make their decisions accordingly. For example, Game Theory assumes that, in the couple setting, nobody refuses the job: indeed, if the female refuses, then the male will maximize his utility by accepting, since he prefers "accepting while the other refuses" over "both refusing"; if the female accepts, then he still maximizes his utility by accepting, since he prefers "both accepting" over "refusing while the other accepts"; The same holds for the female.

Another assumption is that players are rational, that is they know everything about the game. Specifically, every player exactly knows the other components of the system, their strategies, their utility functions, and, finally, she knows that the same information is available to every other player. Even if this assumption can appear to be unrealistic, still it has not been an obstacle to the huge application of Game Theory. Nevertheless, a large literature (Osborne M. J. *et al.*,1994; Fudenberg D. *et al.*,1998; Gigerenzer G. *et al.*, 2002) introduced new tools and concepts that works even when this assumption is weakened.

Given the definition of strategic game and the description of the main assumptions, we are now ready to introduce the most prominent concept in Game Theory, the equilibrium. This is an outcome of the strategic game in which every (rational and utility-maximizer) player cannot improve her own utility by taking an alternative decision (given that the other players do not change their actions). Thus, in the couple example, the "both accepting" outcome is an equilibrium: indeed, as observed above, when the other is accepting, for both the male and the female it is more convenient to accept the job. In Figure 1 the equilibrium corresponds to the only outcome from which there is no outgoing edge.

In other word, in an equilibrium every player is "happier" with their own decision, than if she takes an alternative strategy. Thus, equilibria represent the most probable outcomes in a given strategic game. Indeed, in every

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outcome that is not an equilibrium, there is a player that has an incentive to change her decision, and thus it is highly unlikely that this player will accept to remain in that outcome.

In the couple example discussed above, it is easy to identify the equilibrium, since every player has a strategy that is the best one regardless of what the other players do. However, this is not always the case. Consider indeed the following setting: a boy and a girl (the players) must choose their holidays location among three alternatives: Paris, Mykonos, Ibiza (the strategies). In order to model a game we need to describe the utility functions. Suppose then that the girl always prefers to choose the same location as the boy, and among the three locations she prefers to go to Paris. The boy instead likes to have a romantic trip to Paris with the girl. But, if this is not possible, he would prefer to go to Ibiza or Mykonos. Moreover, he also prefers to enjoy these places alone and not with the girl.

Note that for each player, there is no strategy that is the best one regardless what the other players do. Indeed, for the girl, Paris is the best choice only if also the boy chooses to go to Paris. Otherwise she would maximize her utility by choosing the same location as the one chosen by the boy. Similarly, for the boy, Paris maximizes the utility only if also the girl chooses to go to Paris. Otherwise he would maximize her utility by choosing the location that has not been chosen by the girl. However, still there is an equilibrium. Consider, indeed, the outcome in which both the boy and the girl go to Paris. This is the most preferred outcome for both players, and thus nobody has an incentive to unilaterally move away from this outcome.

A more complex setting is the following: consider two players, a master painter and his best apprentice. Each of them has to choose whether to paint a religious subject or a mundane subject. The master painter prefers that the apprentice will choose the same subject, whereas the apprentice prefers to print different subjects. As above, for both players there is no choice that is the best regardless of the other player's choice. But now the situation is even worse. Indeed, for each possible outcome there is one player that does not like that outcome: if they both select the same subject, the apprentice has an incentive to change; if they select different subjects, then it is the master that would maximize her utility by deviating.

It seems that there is no equilibrium at all in this setting. However, let us suppose that the master and the apprentice are allowed to take the following randomized decision: “I will flip a coin; if heads, I will paint a religious subject, otherwise I will paint a mundane subject". A decision like this one is called mixed strategy, since it allow to mix multiple strategies such that each of them has some chance to be played. One can view mixed strategies as beliefs about what other players will do. For example, if the master does not know what the apprentice will paint, the best guess that he can do is to believe that he will choose the subject at random. At the same time, if the master does not want to give any advantage to the apprentice, then the former must force the latter to guess that he will choose the subject at random.

Assume now that both master and apprentice adopt the mixed strategy described above. Then in half of the cases (i.e., when both the master's and the apprentice's coin land head or both land tail) they paint the same

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subject, and in the other half (i.e., when one coin lands head and the other lands tail) they paint different subjects. Suppose now that one of the players, say the master, deviates from this mixed strategy, that is he chooses to paint a subject with a larger probability. The apprentice will still choose the same subject as the master in half of the cases, and a different subject in the remaining half. Hence, the deviation is not beneficial to the master, since it does not increase his utility. In other words, no player has an incentive to deviate from the mixed strategy, and thus the outcome in which both players adopt that mixed strategy is an equilibrium.

Thus, there are games in which, even if there is no equilibrium in which all players choose a fixed strategy in advance, still there are equilibria in which players adopt mixed strategies. Indeed, a celebrated result by John Nash (Myerson R. B., 2013) claims that for every game, if players are allowed to adopt mixed strategies, then there is at least one equilibrium.

The descriptions in Section 2 and 3 highlights that there are similarities between the setting of strategic inter-dependence of Game Theory and the holistic setting of Systems Theory.

Actually, a more detailed analysis of these similarities highlights a more relevant correspondence, that we will describe in the next section, between the concepts of strategy and equilibrium, and the ones of relation and rule.

This correspondence suggests, in our opinion, that Systems Psychology can import some concepts from Game Theory in order to expand its theoretical background and to introduce new tools in the clinical practice.

4. Where Game Theory meets Systems Psychology

The first connection between Game Theory and Systems Psychology is the correspondence between the parts of a system and the players of a game. The strategies of these players can be surely likened to the types of relation that every part can establish with the other components of a system. For example, Bateson (Bateson G., 1972) states that the human character comes in opposite polarities, e.g., power vs. submission or aid-prone vs. reliance-prone, and that every individual, even if she reveals only one polarity of its character, keeps the ability of behaving even according the opposite polarity.

Anyway, this is not the only role that can be played by strategies in Systems Psychology. Indeed, a prominent observation of Minuchin states within the systems a fundamental role is played by the hierarchical structure of its members. However, some members can refuse to assume their position in the hierarchy. E.g., a family is a hierarchical structure where the parents are the authorities, but it is not rare to see fathers or mothers that do not play this role correctly. It is possible then to model the acceptance or refusal of the role as strategies of an individual.

Note that in both case, it makes sense to talk about mixed strategies. For example, Bateson (Bateson G., 1972) explicitly invites to mix the two polarities in the character. Similarly, it is common for the therapist to meet people that play only partially their role in the hierarchy, and mix their behavior with aspects that mainly concern other roles.

The most inspiring correspondence between Game Theory and Systems

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Psychology is the one between rules and equilibria. Let us recall the definition of these two terms: a rule is described as a kind of interaction between the members of the systems, as long as this interaction turns out to be stable and persistent for long time. An equilibrium is, instead, a profile of strategies of the players, such that no player has an incentive to move away from this profile. Since we likened the kinds of interaction with strategies, it is then evident that the two definition are related. Indeed, stability and persistence are only possible if incentives to deviate are absent for every member of the system. This interpretation is also confirmed by Jackson (Jackson Don D.,1965) that states that a rule must guarantee that each member in the system is able to make profitable the relation with other members.

This correspondence between rules and equilibria appears to be mined by the theory behind the origin of the symptom (Guttman H. A.,1991). It indeed states that the symptom is the result of a tension between the homeostatic propensity to do not change the ongoing rules, and the demand for an alteration of these same rules. This demand is in stark contrast with our hypothesis that rules correspond to equilibria from which nobody has an incentive to deviate. This discrepancy can be settled by interpreting the symptom as an evidence that the game is changed, and, consequently, also its equilibria. This is in line with the assumption that the symptom usually occurs in correspondence of an evolution of the system (Ed Carter B. *et al.*, 1988), caused either by external influences, issued by para-normative events (unemployment, death, etc.), or by internal perturbations, arising in the natural system's life-cycle. According to this interpretation, the demand for alteration is motivated by the fact that the ongoing rules ceases to be equilibria of the game. Not only, even the homeostasis can be easily motivated in this framework: the ongoing rules were an equilibrium in the system before that it evolved, and their conservation is caused by the fact that players do not realize that the game is changed. That is, we are likening this change in the game to a first-order transaction (Sandholm W. H., 2010), that is a transaction in the relation that does not change the followed rules. Note that this framework does not require that the system's members that have an incentive to move away from the ongoing rules are the one for which the symptom occurs. It is indeed possible that these members delegate others as carriers of the symptom, as highlighted through the theory of identified patient (Guttman H. A.,1991).

There is still a difference between the concept of rule and equilibrium. Given the set of players, their strategies and their utility functions, it is then possible to compute the equilibria of the game and to describe their features. However, in the therapeutic practice, the psychologist faces her patients (players), and she is instructed by theory about the kinds of interaction (strategies). But the therapist investigates about the rules (equilibria), without the need to define utility functions.

That is, utility functions appear to play no role in the therapeutic practice. However, the correspondence between rules and equilibria suggests an alternative explanation of this difference: the therapist is simply approaching the game from an opposite direction with respect to the game-theorist. Whereas the latter needs to compute the equilibria of the given

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game, the psychologist uses the equilibria to learn the game, and, in particular, to build the utility functions. Indeed, the role of the therapy consists in driving the system so that it reacts to the changes in the relation among its members. That is, the psychologist enables a second-order transaction (Guttman H. A., 1991), that requires a transformation in the rules of the system that mirrors the change in the relation. However, the therapist can effectively operate this transaction only by learning how the system works.

This viewpoint then allows to the vast theory of equilibria in Game Theory (Fudenberg D., *et al.*,1991; Von Neumann J. *et al.*,1944; Moreno D. *et al.*,1996; Leyton-Brown K. *et al.*, 2008), to be a useful tool that psychologists can use in therapy. We show this, by reviewing a clinical vignette under the lens of Game Theory. It is a classical example that we quote from (Von Bertalanffy L., 1969): in a couple, the wife complains about the husband being illiterate, that forced her to bear the responsibility in the family.

This first description is sufficient to model a game. The husband and the wife are the two players. The husband has two strategies: he can either continue to be illiterate, or to learn to read and write. Clearly, illiteracy is a metaphor for a submission polarity in his character. Similarly, the wife has two strategies: she can bear responsibility, or escape from this role. The ongoing rule in the couple consists in an outcome that exhibits a complementary relation (Bateson G., 1972), where the wife is the powerful member and the husband the submitted part. But this is no longer an equilibrium, since the wife designs the husband as carrier of the symptom.

As described above, in the case of an identified patient, is usually the member that point to the symptom to have an incentive to deviate from the ongoing rule. Hence, we achieve the utility function of the wife when the husband is illiterate: she prefers to do not bear the responsibility. Then, the ongoing rule is no longer an equilibrium, and only the homeostatic trend keeps this rule. As for the husband's utility, since his illiteracy reveals a submission polarity, we have that, as long as he is illiterate, he would prefer to be submitted by the wife (that is, he prefers she bears the responsibility). Note that the outcome in which he is illiterate, but the wife escapes from her role cannot be an equilibrium. Indeed, if the husband does not find anyone responsible, he is forced to assume these responsibilities on itself, and thus to study. We summarize these findings in Figure 2.

This analysis teaches to the therapist which action to take for enabling a transformation in the rules of the system that makes them to correspond to the new equilibrium. Indeed, as discussed, every equilibria of the game must involve a change in the strategy of the husband, and the duty of the therapist is to enable this change.

This is exactly what happened in the case that we are reviewing: at some time, the husband learns to read and write. That is, he dismisses his submission polarity and accepts to take the responsibilities implied by his role. This ceases the homeostatic trend that is blocking the game from reaching his new equilibrium. However, this does not mean that the outcome that has been reached is an equilibrium. Indeed, what happened has been that the wife asked for a divorce. This is because the wife still prefers a complementary relation in which she is the powerful part. Hence,

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when the husband “steals" her power, she is not happy and prefers to escape away. This situation is depicted in Figure 3.
This analysis done through the game-theoretic lens highlights how the divorce is the only possible outcome that may be reached. Hence, the therapy is not failed, since it successfully drove the system in the new equilibrium.
We hope that this example enlightens the correspondence that we drew above, and highlights how this correspondence can be useful to psychologists in their therapeutic practice.

Conclusions

In this work we draw a connection between Game Theory and Systems Psychology. To this aim we reviewed some of the basics concepts in Game Theory and analyzed the correspondence between these concepts and similar concepts arising in Systems Theory.
However, we believe that the relationship between these two disciplines is even deeper. Indeed, there are some findings in Systems Psychology, such as, for example, the schismogenesis (Bateson G., 1972), that can be easily explained through a game-theoretical approach.
Moreover, game-theorists introduced many concepts besides the one described in Section 3, and some of these can be of interest for the Systems Psychology. For example, the equilibrium concept assumes that no player has an incentive to unilaterally deviate from the given outcome. Thus, it discards the possibility that players can create coalitions, whose members jointly deviate and improve their welfare. However, there is a rich literature (Simon F. B. *et al.*,1985; Bernheim B. D. *et al.*,1987; Minuchin S.,1974; Driessen T., 1988) that tries to address this issue. It would be interesting to relate the concepts in these works with the one of alliance introduced by Whitaker (Watzlawick P. *et al.*, 2011).
Another limit of strategic games as we described in Section 3 is that they do not consider the dynamical aspects behind the evolution of a system. These dynamical aspects are instead particularly relevant in Systems Psychology, as highlighted by Bowen (Bowen M. *et al.*,1979). However these dynamical aspects have been analyzed in the game-theoretic literature (Fudenberg D., *et al.* 1998; Rubinstein A., 1997), and it would be interesting to understand if their findings can be useful for the therapeutic practice.
It would be also interesting to analyze if other connections can be found with those concepts that weaken the assumption of rationality (Osborne M. J. *et al.*, 1994; Fudenberg D. *et al.*, 1998; Gigerenzer G. *et al.*, 2002).
Finally, in Game Theory there is also a rich literature (Fudenberg D. *et al.*,1991) that highlights how an external third party can change the equilibria of a game or create new equilibria. The concepts and the results in this literature may be of interest for psychologists. They can be used for avoiding bad equilibria (such as the divorce in the example above) and for explaining the effectiveness of many of the used tools.

Figures

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An example of game

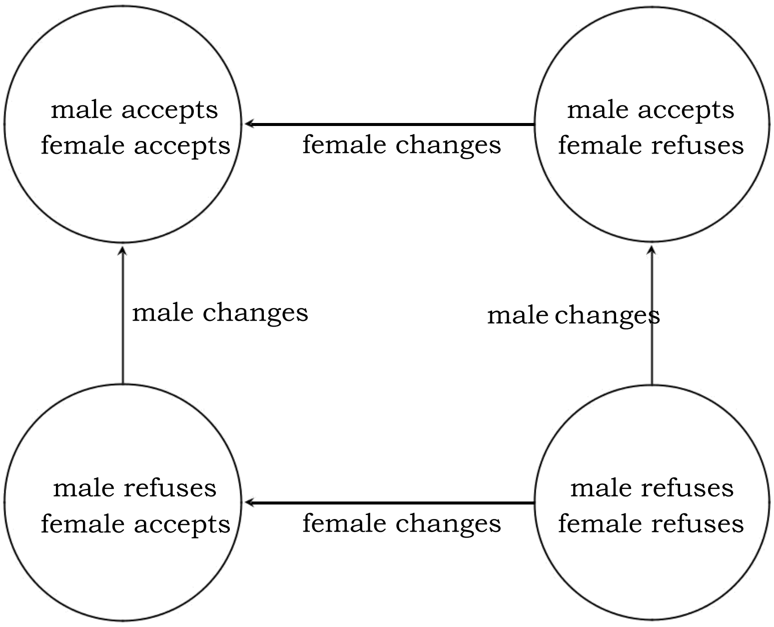

Figure 1: An alternative way of representing a game: every circle is a possible outcome of the game, and an edge from a circle to another one means that the corresponding player prefers to change her strategy from the one that she plays in the former circle to the one she plays in the second circle.

Clinical case: the game before the therapy

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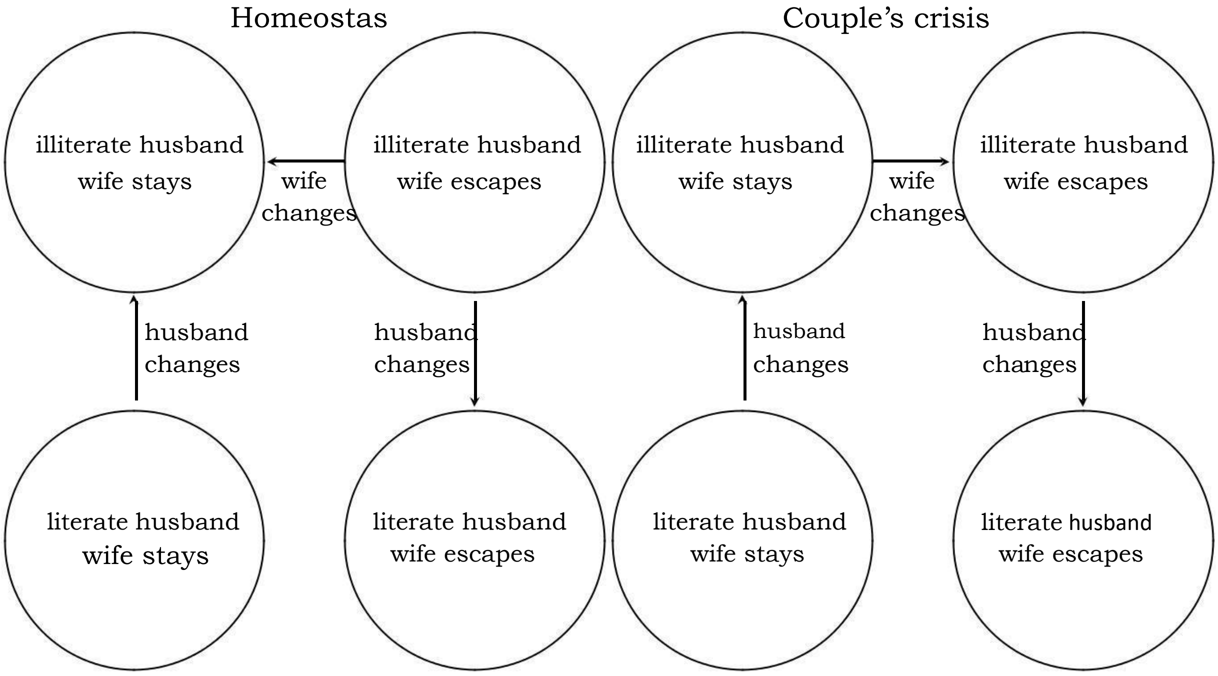

Figure 2: On the left it is pictured the game that players played in the past, in which the state where the husband is illiterate and the wife bears the responsibility is an equilibrium (since the corresponding circle has no outgoing edges). On the right, instead, there is the new game that booted the couple's crisis. Note that in this new game, the preference of the female is changed, and, consequently, the old equilibrium is no longer an equilibrium. Similarly, the state where the husband is illiterate and the wife escapes is not an equilibrium. Observe that both games are only partially defined, since it is not known which outcome is preferred by the wife when the husband decides of learning to read and write. In particular, this means that it is not possible to understand which is the equilibrium in the new game.

Clinical case: the game after the therapy

During the therapy After the therapy

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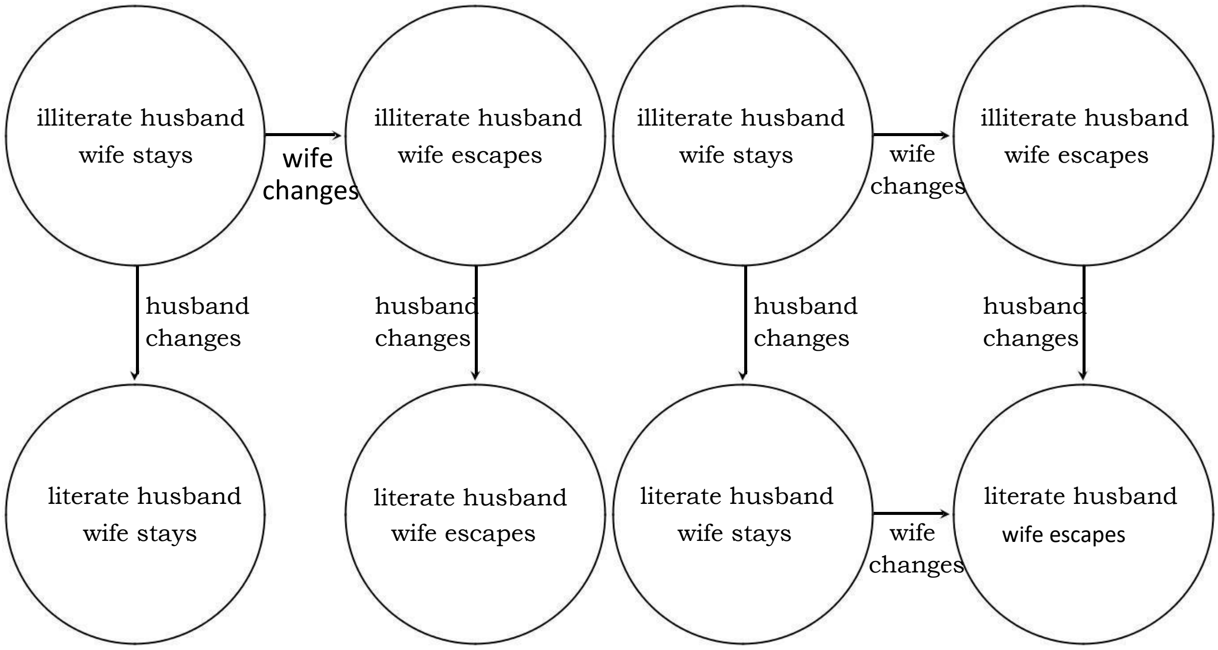

Figure 3: On the left it is pictured the game resulting from the intervention of the therapist. This intervention corresponds to change the preference of the husband from being illiterate to being literate (that is, it changes the direction of the first vertical edge). Still, this is a partial definition of the game, since the preference of the wife when the husband is literate is not known. When, after the therapy, this preference becomes known, then the game can be fully represented, as we did on the right. Here, one can check that the only outcome of this game that is in equilibrium is the one in which the husband is literate and the wife escapes, since the corresponding circle is the only one without outgoing edges.

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